SYNOPSIS

RANKING FUZZY NUMBERS AND THEIR APPLICATIONS TO FUZZY DECISION-MAKING PROBLEMS

Submitted to

BINOD BIHARI MAHTO KOYALANCHAL UNIVERSITY, DHANBAD, JHARKHAND, INDIA

For

Research Work in

MATHEMATICS

By

SHATABDI SINHA

(NET Research Scholar)

Under the Supervision of

Dr. Shiv Prasad
Assistant Professor
Department of Mathematics
Binod Bihari Mahto Koyalanchal University, Dhanbad.

April - 2024

Shatabdi Sinha
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CERTIFICATE

This is to certify that the Synopsis entitled "**Ranking Fuzzy Numbers and their Applications to Fuzzy Decision-making Problems**" is being submitted to Binod Bihari Mahto Koyalanchal University, Dhanbad by **Miss Shatabdi Sinha** for research work for the award of the degree of Doctor of Philosophy in Mathematics is under my supervision.

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SYNOPSIS

Ranking Fuzzy Numbers and their Applications to Fuzzy Decision-making Problems

Introduction

In classical sets, every member has a common membership grade equal to one, which approves only their presence in the set, whereas in a fuzzy set, each member has its membership grade lying in the closed unit interval $[0, 1]$. The membership grade characterizes the degree of importance of the member in the set. Thus, a fuzzy set $\tilde{A}$ defined in the universe of discourse X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x) \mid x \in X)\}$$

where $\mu_{\tilde{A}}(x)$ is known as the membership grade of x in $\tilde{A}$; its value lies in the unit interval $[0, 1]$, and characterizes the degree of importance of x in the set $\tilde{A}$.

Fuzzy sets were first described by Zadeh [1]. He expanded the set-theoretic concepts like complementation, union, intersection, relation, convexity, etc. to fuzzy sets, and demonstrated numerous features of these conceptions in the framework of fuzzy sets. Even though Fuzzy Set Theory is a young theory, whose age is only 59; it has covered a lot of ground by today.

In practical daily life, one frequently uses expressions such as “hot”, “cold”, “about there”, and so on, to convey a real notion in a vague form. In general, such expressions are termed as imprecise or ill-defined. Every individual imparts a distinctive meaning to these expressions. In mathematics, it is crucial to make such vague notions operational. To this end, one can introduce well-defined mathematical perceptions of vague notions that satisfactorily impart intuitive meaning for everybody. To cope with imprecise numerical quantities practically, Zadeh introduced fuzzy numbers as a fuzzy subset of the real line “$\mathbb{R}$”, characterized by convex and piecewise continuous membership functions as follows:

A fuzzy subset $\tilde{A} = \{(x, \mu_{\tilde{A}}(x) \mid x \in \mathbb{R})\}$ of the set $\mathbb{R}$ of real numbers is termed a fuzzy number, provided the membership function $\mu_{\tilde{A}}(x)$: $\mathbb{R} \longrightarrow [0, 1]$ that grades $x \in \mathbb{R}$, fulfill the following conditions for $a, b, c, d \in \mathbb{R}$, $(a \leq b \leq c \leq d)$,

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(i) $\mu_{\tilde{A}}(x)$ is a piece-wise continuous mapping from $\mathbb{R}$ onto the unit interval $[0,1]$,

(ii) $\mu_{\tilde{A}}(x) = 0,\ \forall\ x \in]-\infty, a]$,

(iii) $\mu_{\tilde{A}}(x)$ is continuously increasing on $[a,b]$,

(iv) $\mu_{\tilde{A}}(x) = 1,\ \forall\ x \in [b,c]$,

(v) $\mu_{\tilde{A}}(x)$ is continuously decreasing on $[c,d]$,

(vi) $\mu_{\tilde{A}}(x) = 0,\ \forall\ x \in [d,\infty[$.

The fuzzy number is conveniently represented by $\tilde{A} = (a,b,c,d)$, and its membership grading $\mu_{\tilde{A}}(x)$ is stated as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{A}}^{L}(x)\,; & x \in [a,b] \\ 1\,; & x \in [b,c] \\ \mu_{\tilde{A}}^{R}(x)\colon & x \in [c,d] \\ 0; & \text{otherwise} \end{cases}$$

where $\mu_{\tilde{A}}^{L}(x)\colon [a,b] \longrightarrow [0,1]$ is continuously increasing, called the left membership function and $\mu_{\tilde{A}}^{R}(x)\colon [c,d] \longrightarrow [0,1]$ is continuously decreasing, called the right membership function of $\tilde{A} = (a,b,c,d)$.

Dubois and Prade [7] developed the notion of a fuzzy number and the corresponding fuzzy arithmetic.

The fuzzy number ordering is a process to define a real-valued function that maps each fuzzy number into the real line. Fuzzy numbers have this crucial feature, making them suitable for risk analysis and decision-making in real-world situations like Medical Science, Engineering Designs, Data Analysis (DA), Control, Optimization, Financial Management, and so on. Decision-making is a logical process to identify the best alternative in accordance with the values and preferences of a decision-maker. Therefore, decision-making plays a vital role, especially in industrial and service institutions for minimizing their input cost and time, and maximizing the profit while improving the quality of outputs. But, in real-life scenarios, the decision-making generally faces a lot of confusion and ambiguity because of the involvement of uncertainty with the alternatives. To deal with such kind of vagueness in human thought, the fuzzy set theory plays a crucial role and makes such vague notions operational by representing them in the terminology of fuzzy sets. Ranking of fuzzy numbers ensures their applications in choosing the best alternative in the fuzzy environment.

The pioneering work of ranking imprecise quantities characterized as fuzzy sets for selecting an optimal alternative was first proposed by Jain [6].

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Literature Review

After the pioneering work of ranking proposed by Jain [6], Yager [8] evolved the theory of ranking fuzzy numbers based on the centroid point; Bortolan and Degani [9] assessed and summarized a few other significant methods. Since then, the literature has acknowledged a large number of suggestions to rank fuzzy numbers using various notions. The six major categories into which these methods can be classified are based on centroid, distance, area, lexicography, the left and right spreads, and lastly the decision maker's viewpoint.

Yager [8] is credited with one of the earliest works on fuzzy subset ordering based on the centroid value or the weighted mean value on the horizontal axis defined as:

$\bar{x} = \frac{\int_{\tilde{A}} x f_{\tilde{A}}(x) dx}{\int_{\tilde{A}} f_{\tilde{A}}(x) dx}$, where $f_{\tilde{A}}(x)$ denotes the reference function of the fuzzy number $\tilde{A}$. If the fuzzy number possesses a higher weighted mean value, then it is considered to be larger.

Cheng [14] devised a distance approach defined as $R(\tilde{A}) = \sqrt{\bar{x}^2 + \bar{y}^2}$, which ranks the fuzzy numbers as per the distance of its centroid from the original point, where $(\bar{x}, \bar{y})$ represents the coordinates of the centroid point of the fuzzy number. If the distance is larger, the fuzzy number is considered to be larger.

Chu and Tsao [15] initially examined the drawbacks of the ranking techniques introduced by Cheng [14], followed by another approach in which they proposed the area ranking method by defining the suggested area between centroids of fuzzy numbers from the original point as $S(\tilde{A}) = \bar{x}\,\bar{y}$. If the area is larger, then the fuzzy number is considered to be larger. Using several comparative examples, the advantages of the suggested technique were demonstrated.

Among the decision maker's perspective-based methods, Liou and Wang [11] introduced a method of ordering fuzzy numbers using the total integral value of the fuzzy number, defined as $I_T^{\alpha}(\tilde{A}) = \alpha I_R(\tilde{A}) + (1-\alpha) I_L(\tilde{A})$, where $\alpha \in [0,1]$ is known as the indicator of optimism, which represents the viewpoint of a decision-maker. $I_R(\tilde{A})$ and $I_L(\tilde{A})$ are the right and left area's integral to the fuzzy number. The method works regardless of the normality and the type of the membership functions. To address the limitations of the method based on the viewpoint of the decision maker, Yu and Dat [22] and Nguyen [25] suggested revised ranking approaches. Yu and Dat [22] proposed a novel integral value to the fuzzy

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number as $S_T^{\alpha} = \alpha S_R(\tilde{A}) + (1-\alpha) S_L(\tilde{A})$, where $S_R(\tilde{A})$ and $S_L(\tilde{A})$ are the left and right areas of the fuzzy number from the point x_{min}, where $x_{min} = \min\{a, b, c, d\}$; $\tilde{A} = \{a, b, c, d\}$. Nguyen [25] demonstrated a unified index that multiplies two different discriminatory components, the weighted mean and the weighted area of the fuzzy number, where the weighted area is defined as a convex combination of the areas described by the left-right reference functions of the fuzzy number from the membership axis.

Among the left and right spread-based methods, to find the ordering value of each fuzzy number, Chen [10] introduced the idea of maximizing and minimizing sets. The order of n fuzzy numbers is determined using the total utility evaluated using the left and right utility values obtained.

Fuzzy decision-making was first proposed by Lotfi Zadeh (co-authored with Rechard Wellman). The paper [5] of Zadeh is a rich source of ideas regarding fuzzy decision-making and fuzzy optimization.

Problem to be Investigated

In the proposed work, we will investigate the novel ranking indexes to rank the fuzzy numbers with linear and non-linear membership functions as well as other general types of fuzzy numbers, based on centroid point, left-right spreads, areas described by the membership functions, angles of the reference functions, and the perspective of a decision maker. Comparative studies will be performed with different representative approaches using several fuzzy-number examples from the literature. We shall also investigate the applications of the ranking indexes to fuzzy decision-making processes, fuzzy optimization techniques, and fuzzy risk analysis problems.

Objective

To demonstrate appropriate approaches and significant methods for ordering fuzzy numbers and their applications to fuzzy decision-making problems related to the real world along with a study of comparative examples.

Hypothesis

The fuzzy numbers represent ambiguous data values, so it is challenging to order them based on their magnitude. Thus, it is essential to rank the fuzzy numbers for making a decision. Since alternatives are often measured by fuzzy numbers in a fuzzy environment, comparing the fuzzy numbers actually means comparison of the alternatives. There are numerous types of methods based on different parameters of the fuzzy number for ranking that exist in the literature; but in terms of discrimination and consistency, any particular method cannot be regarded as more advantageous than others, which yields scope for further studies to develop novel ideas for fuzzy number ranking and their applications to decision making and risk analysis problems.

Methodology

A systematic review of the literature (e-papers, e-journals & e-proceedings) available in open educational resources and online scientific databases is intended using the internet services on portable computing devices. Efforts will be made to obtain permission to access library facilities at other institutions. Membership of educational library portals will be registered and subscribed as and when required for research problem investigation. Participation and presentation of research manuscripts in seminars/ conferences within the scope will be of keen interest.

Research type

The discussions proposed to be included in the thesis will be mainly those of theoretical techniques, with suitable numerical examples that are common for extensive comparative studies related to various types of fuzzy numbers.

Significance

Fuzzy Set Theory provides fuzzy numbers as an effective tool for representing vague concepts of information. The importance of prioritizing fuzzy numbers in practice cannot be overstated because the concept of the best choice entirely relies on comparison. Therefore, how to make a preference for fuzzy numbers is a major

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issue. The ordering of fuzzy numbers is a relevant feature for their use in real-life linguistic decision-making and risk analysis such as in Crisis Management, Medical Science, Engineering Designs, Data Analysis (DA), Fuzzy control, Optimization techniques, and various applied models.

Chapterization

Our research study is proposed to comprise four chapters. Chapter I shall present a general introduction and literature review followed by Chapter II pertaining to the relevant definitions, properties, and some remarks. Chapter III will comprise novel fuzzy ranking approaches with comparative studies of some representative approaches using several fuzzy-number examples from the literature that are common for a broad range of numerical studies. Chapter IV will consist of the applications of ranking fuzzy numbers in analytical problems and fuzzy decision-making.

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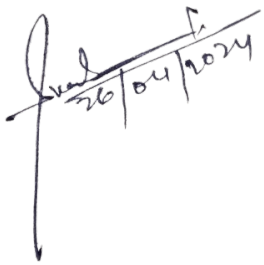

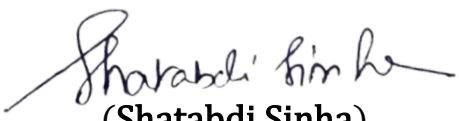

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